Unit 01

MATRICES AND DETERMINANTS

Idea of matrices:

The idea of matrices was given by Arthur Cayley, an English mathematician of nineteenth century who first developed, "Theory of Matrices" in 1858.

Q1. Define the following terms.

(i) Matrix

"A rectangular array or a formation of a collection of real numbers, say 0, 1, 2, 3, 4 and 7, such as: $\begin{matrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{matrix}$ and then enclosed by brackets '[ ]' is said to form a matrix $\begin{bmatrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{bmatrix}$. Similarly $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$ is another matrix.

The matrices are denoted conventionally by the capital letters A,B,C,....,M,N etc. of the English alphabet.

(ii) Order of a Matrix

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, m-by-n, For example, $M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ is order 2-by-3,

(iii) Equal Matrices

Let A and B be two matrices. Then A is said to be equal to B, and is denoted by A = B, if and only if;

(i) The order of A = The order of B

(ii) Their corresponding entries are equal.

Examples

(i) $A = \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$

are equal matrices.

We see that:

(a) The order of matrix A = The order of matrix B

(b) Their corresponding elements are equal.

Thus A = B

(ii) $L = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $M = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ are not equal matrices.

We see that: order of L = order of M but entries in the second row and second column are not same, so $L \neq M$.

(iii) $P = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$

are not equal matrices.

We see that order of P ≠ order of Q, so $P \neq Q$.

Exercise 1.1

1. Find the order of the following matrices.

$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, order of A is 2-by-2

$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, order of B is 2-by-2

$C = \begin{bmatrix} 2 & 4 \end{bmatrix}$ order of C is 1-by-2

$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, order of D is 3-by-1

$$E=\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix},$$ order of E is 3-by-2

$$F=[2]$$ order of F is 1-by-1

$$G=\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$,order of G is 3-by-3

$$H=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$$ order of H is 2-by-3

2. Which of the following matrices are equal?

$A=[3]$,

$$B=[3 \quad 5], C=[5-2]$$

$$D=[5 \quad 3], E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix},$$

$$F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix},$$

$$H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}, I=[3 \quad 3+2]$$

$$J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$$

Ans. Equal matrices are

$A=C$ $B=I$

$E=H=J$ $F=G$

3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix}=\begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Ans. $a+c=0$ ....(i)

$a+2b=-7$ ....(ii)

$c-1=3$ ....(iii)

$4d-6=2d$ ......(iv)

From (iii)

$c=3+1$

$\boxed{c=4}$

From (iv)

$4d-2d=6$

$2d=6$

$d=\frac{6}{2}$

$\boxed{d=3}$

Put value of $c=4$ in (i)j

$a+4=0$

$\boxed{a=-4}$

Put value of $a=-4$ in (ii)

$-4+2b=-7$

$2b=-7+4$

$2b=-3$

$\boxed{b=\frac{-3}{2}}$

Types of Matrices

(e) Row Matrix.

A matrix is called a row matrix if it has only one row.

e.g.; the matrix $M=[2 \quad -1 \quad 7]$ is a row matrix of order 1-by-3 and $M=[1 \quad -1]$ is a row matrix of order 1-by-2.

(ii) Column Matrix.

A matrix is called a column matrix if it has only one column e.g., $M=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

and $N = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ are column matrices of order 2-by-1 and 3-by-1 respectively.

(e) **Rectangular Matrix.**

A matrix is called rectangular if, the number of rows of M is not equal to the number of columns of M.

e.g. $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$;

$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$; $C = [1 \quad 2 \quad 3]$ and

$D = \begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$ are all rectangular matrices. The order of A is 3-by-2, the order of B is 2-by-3, the order of C is 1-by-3 and order of D is 3-by-1, which indicates that in each matrix the number of rows ≠ the number of columns.

(e) **Square Matrix.**

A matrix is called a square matrix if its number of rows is equal to its number of columns.

e.g., $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$ and $C = [3]$ are square matrices of orders 2-by-2, 3-by-3 and 1-by-1 respectively.

(v) **Null or Zero Matrix.**

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g., $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $[0 \quad 0]$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

and $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are null matrices of orders 2-by-2, 1-by-2, 2-by-1, 2-by-3 and 3-by-3 respectively. Null matrix is represented by O.

(vi) **Transpose of a Matrix.**

A matrix obtained by interchanging the rows into columns or columns into rows of a matrix is called transpose of that matrix. If A is a matrix then its transpose is denoted by A^t.

e.g., (i) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 4 & -2 \end{bmatrix}$

, then $A^t = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 3 & 0 & -2 \end{bmatrix}$

Note: *If a matrix A is of order 2-by-3 then order of its transpose A^t is 3-by-2*

(vii) **Negative of a Matrix.**

Let A be a matrix. Then its negative, –A, is obtained by changing the signs of all the entries of A, i.e.,

If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$.

(viii) **Symmetric Matrix.**

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric if $A^t = A$.

e.g. (i) If $M=\begin{bmatrix}1&2&3\\2&-1&4\\3&4&0\end{bmatrix}$

is a square matrix, then

$M^t=\begin{bmatrix}1&2&3\\2&-1&4\\3&4&0\end{bmatrix}=M$. Thus M is a symmetric matrix.

(ii) If $A=\begin{bmatrix}2&1&3\\-1&2&2\\3&1&3\end{bmatrix}$,

then $A^t=\begin{bmatrix}2&-1&3\\1&2&1\\3&2&3\end{bmatrix}\neq A$

Hence A is not a symmetric matrix.

(x) Skew-Symmetric Matrix.

A square matrix A is said to be skew-symmetric if $A^t=-A$.

e.g., If $A=\begin{bmatrix}0&2&3\\-2&0&1\\-3&-1&0\end{bmatrix}$, then

$$A^t=\begin{bmatrix}0&-2&-3\\2&0&-1\\3&1&0\end{bmatrix}=\begin{bmatrix}0&-2&-3\\-(-2)&0&-1\\-(-3)&-(-1)&0\end{bmatrix}$$

$$=-\begin{bmatrix}0&2&3\\-2&0&1\\-3&-1&0\end{bmatrix}=-A$$

Since $A^t=-A$, therefore A is a skew-symmetric matrix.

(x) Diagonal Matrix.

A square matrix A is called a diagonal matrix if atleast any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

e.g., $A=\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$, $B=\begin{bmatrix}1&0&0\\0&2&0\\0&0&2\end{bmatrix}$

and $C=\begin{bmatrix}0&0&0\\0&1&0\\0&0&3\end{bmatrix}$ are all diagonal matrices of order 3-by-3.

$M=\begin{bmatrix}2&0\\0&3\end{bmatrix}$ and $N=\begin{bmatrix}1&0\\0&4\end{bmatrix}$

are diagonal matrices of order 2-by-2.

(xi) Scalar Matrix.

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero. For example $\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}$

where k is a constant $\neq 0, 1$.

Also $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$, $B=\begin{bmatrix}3&0\\0&3\end{bmatrix}$ and

$C=[5]$ are scalar matrices of order 3-by-3, 2-by-2 and 1-by-1 respectively.

(xii) Identity Matrix.

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I.

e.g., $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ is a 3-by-3 identity matrix.

$B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ is a 2-by-2 identity matrix.

$C=[1]$ is a 1-by-1 identity matrix.

Exercise 1.2

1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

Ans. $A=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, Null matrix

$B=\begin{bmatrix}2 & 3 & 4\end{bmatrix}$, Row matrix

$C=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix}$, Column matrix

$D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, Unit matrix

$E=\begin{bmatrix}0\end{bmatrix}$, Null matrix

$F=\begin{bmatrix}5\\ 6\\ 7\end{bmatrix}$ Column matrix

2. From the following matrices, identify

(a) Square matrices
(b) Rectangular matrices
(c) Row matrices
(d) Column matrices
(e) Identity matrices
(f) Null matrices

Ans. (a) Square Matrices:

(iii) $\begin{bmatrix}6 & -4\\ 3 & -2\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(viii) $\begin{bmatrix}1 & 2 & 3\\ -1 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$

Ans. (b) Rectangular Matrices:

(i) $\begin{bmatrix}-8 & 2 & 7\\ 12 & 0 & 4\end{bmatrix}$

(ii) $\begin{bmatrix}3\\ 0\\ 1\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$

Ans. (c) Row Matrices:

(vi) $\begin{bmatrix}3 & 10 & -1\end{bmatrix}$

Ans. (d) Column Matrices:

(ii) $\begin{bmatrix}3\\ 0\\ 1\end{bmatrix}$

(vii) $\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$

Ans. (e) Identity Matrices:

(iv) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Ans. (f) Null matrices:

(ix) $\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$

3. From the following matrices, identify diagonal, scalar and unit (identity) matrices.

$$A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix},$$

$$B=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$D=\begin{bmatrix}3 & 0\\ 0 & 0\end{bmatrix},$$

$$E=\begin{bmatrix}5-3 & 0\\ 0 & 1+1\end{bmatrix}$$

Ans. Scalar matrices:

$$A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix},\ E=\begin{bmatrix}5-3 & 0\\ 0 & 1+1\end{bmatrix}$$

Unit Matrices:

$$C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Diagonal Matrices:

$$A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix},\ B=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},D=\begin{bmatrix}3 & 0\\ 0 & 0\end{bmatrix}$$

$$E=\begin{bmatrix}5-3 & 0\\ 0 & 1+1\end{bmatrix}$$

4. Find negative of matrices A, B, C, D and E when:

$$A=\begin{bmatrix}1\\ 0\\ -1\end{bmatrix},$$

$$B=\begin{bmatrix}3 & -1\\ 2 & 1\end{bmatrix}$$

$$C=\begin{bmatrix}2 & 6\\ 3 & 2\end{bmatrix},\ D=\begin{bmatrix}-3 & 2\\ -4 & 5\end{bmatrix},$$

$$E=\begin{bmatrix}1 & -5\\ 2 & 3\end{bmatrix}$$

Negative of matrices

Ans. $-A=\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix},$

Ans. $-B=\begin{bmatrix}-3 & 1\\ -2 & -1\end{bmatrix}$

Ans. $-C=\begin{bmatrix}-2 & -6\\ -3 & -2\end{bmatrix}$

Ans. $-D=\begin{bmatrix}3 & -2\\ 4 & -5\end{bmatrix},$

Ans. $E=\begin{bmatrix}-1 & 5\\ -2 & -3\end{bmatrix}$

5. Find the transpose of each of the following matrices:

Ans. (i)

$$A=\begin{bmatrix}0\\ 1\\ -2\end{bmatrix},\Rightarrow\ A^t=\begin{bmatrix}0 & 1 & -2\end{bmatrix}$$

$$B=\begin{bmatrix}5 & 1 & -6\end{bmatrix}\Rightarrow B^t=\begin{bmatrix}5\\ 1\\ -6\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 2\\ 2 & -1\\ 3 & 0\end{bmatrix}\Rightarrow C^t=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$$

$$D=\begin{bmatrix}2&3\\0&5\end{bmatrix} \Rightarrow D^t=\begin{bmatrix}2&0\\3&5\end{bmatrix}$$

$$E=\begin{bmatrix}2&3\\-4&5\end{bmatrix} \Rightarrow E^t=\begin{bmatrix}2&-4\\3&5\end{bmatrix}$$

$$F=\begin{bmatrix}1&2\\3&4\end{bmatrix} \Rightarrow F^t=\begin{bmatrix}1&3\\2&4\end{bmatrix}$$

6. Verify that if $A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$, $B=\begin{bmatrix}1&1\\2&0\end{bmatrix}$, **then**

(i) $(A^t)^t=A$

(ii) $(B^t)^t=B$

Ans. (i) $(A^t)^t=A$

L.H.S $=(A^t)^t$

$$A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$$

$$\Rightarrow \quad A^t=\begin{bmatrix}1&0\\2&1\end{bmatrix}$$

$$(A^t)^t=\begin{bmatrix}1&2\\0&1\end{bmatrix}$$

$$(A^t)^t=A=\text{R.H.S.}$$

Hence L.H.S = R.H.S.

Ans. (ii) $(B^t)^t=B$

L.H.S $=(B^t)^t$

$$B=\begin{bmatrix}1&1\\2&0\end{bmatrix}$$

$$B^t=\begin{bmatrix}1&2\\1&0\end{bmatrix}$$

$$(B^t)^t=\begin{bmatrix}1&1\\2&0\end{bmatrix}$$

$$(B^t)^t=B$$
$$=\text{R.H.S}$$

Hence L.H.S = R.H.S.

Addition and Subtraction of Matrices

Define Addition of Matrices.

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

e.g., $A=\begin{bmatrix}2&3&0\\1&0&6\end{bmatrix}$ and $B=\begin{bmatrix}-2&3&4\\1&2&3\end{bmatrix}$ are conformable for addition.

Addition of A and B, written A+ B is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B.

e.g., $A+B=\begin{bmatrix}2&3&0\\1&0&6\end{bmatrix}+\begin{bmatrix}-2&3&4\\1&2&3\end{bmatrix}$

$$=\begin{bmatrix}2+(-2)&3+3&0+4\\1+1&0+2&6+3\end{bmatrix}=\begin{bmatrix}0&6&4\\2&2&9\end{bmatrix}$$

Define Subtraction of Matrices.

If A and B are two matrices of same order then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by A – B.

e.g., $A=\begin{bmatrix}2&3&4\\1&5&0\end{bmatrix}$ and $B=\begin{bmatrix}0&2&2\\-1&4&3\end{bmatrix}$ are conformable for subtraction.

i.e., $A-B=\begin{bmatrix}2&3&4\\1&5&0\end{bmatrix}-\begin{bmatrix}0&2&2\\-1&4&3\end{bmatrix}$

$$=\begin{bmatrix}2-0&3-2&4-2\\1-(-1)&5-4&0-3\end{bmatrix}=\begin{bmatrix}2&1&2\\2&1&-3\end{bmatrix}$$

Multiplication of a Matrix by a Real Number

Let A be any matrix and the real number k be a scalar. Then the scalar multiplication of matrix A with k is obtained by multiplying each entry of matrix A with k. It is denoted by kA.

Let $A=\begin{bmatrix}1 & -1 & 4\\ 2 & -1 & 0\\ -1 & 3 & 2\end{bmatrix}$ be a matrix of order 3-by-3 and k=−2 be a real number.

Then

$$kA=(-2)A$$

$$=(-2)\begin{bmatrix}1 & -1 & 4\\ 2 & -1 & 0\\ -1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}(-2)1 & (-2)(-1) & (-2)(4)\\ (-2)(2) & (-2)(-1) & (-2)(0)\\ (-2)(-1) & (-2)(3) & (-2)(2)\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 2 & -8\\ -4 & 2 & 0\\ 2 & -6 & -4\end{bmatrix}$$

Commutative and Associative Laws of Matrices

(a) Commutative Law under Addition

If A and B are two matrices of the same order, then A + B = B + A is called commutative law under addition.

Let $A=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}$,

$B=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$

Then

$$A+B=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$$

$$=\begin{bmatrix}2+3 & 3-2 & 0+5\\ 5-1 & 6+4 & 1+1\\ 2+4 & 1+2 & 3-4\end{bmatrix}=\begin{bmatrix}5 & 1 & 5\\ 4 & 10 & 2\\ 6 & 3 & -1\end{bmatrix}$$

Similarly

$$B+A=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}+\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5 & 1 & 5\\ 4 & 10 & 2\\ 6 & 3 & -1\end{bmatrix}$$

Thus the commutative law of addition of matrices is verified.

$$A + B = B + A$$

(b) Associative Law under Addition

If A, B and C are three matrices of same order, such that (A+B)+C=A+(B+C) is called associative law under addition.

Let $A=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}$,

$B=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$

and

$C=\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}$

Then

$$(A+B)+C=\left(\begin{bmatrix}2&3&0\\5&6&1\\2&1&3\end{bmatrix}+\begin{bmatrix}3&-2&5\\-1&4&1\\4&2&-4\end{bmatrix}\right)$$

$$+\begin{bmatrix}1&2&3\\-2&0&4\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}2+3&3-2&0+5\\5-1&6+4&1+1\\2+4&1+2&3-4\end{bmatrix}+\begin{bmatrix}1&2&3\\-2&0&4\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}5&1&5\\4&10&2\\6&3&-1\end{bmatrix}+\begin{bmatrix}1&2&3\\-2&0&4\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}6&3&8\\2&10&6\\7&5&-1\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2&3&0\\5&6&1\\2&1&3\end{bmatrix}$$

$$+\left(\begin{bmatrix}3&-2&5\\-1&4&1\\4&2&-4\end{bmatrix}+\begin{bmatrix}1&2&3\\-2&0&4\\1&2&0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2&3&0\\5&6&1\\2&1&3\end{bmatrix}+\begin{bmatrix}3+1&-2+2&5+3\\-1-2&4+0&1+4\\4+1&2+2&-4+0\end{bmatrix}$$

$$=\begin{bmatrix}2&3&0\\5&6&1\\2&1&3\end{bmatrix}+\begin{bmatrix}4&0&8\\-3&4&5\\5&4&-4\end{bmatrix}$$

$$=\begin{bmatrix}6&3&8\\2&10&6\\7&5&-1\end{bmatrix}$$

Thus the associative law of addition is verified:

$$(A+B)+C = A+(B+C)$$

Additive Identity of a Matrix

If A and B are two matrices of same order such that $A + B = A = B + A$ then matrix B is called additive identity of matrix A.

For any matrix A and zero matrix O of same order, O is called additive identity of A as

$A + O = A = O + A$

e.g., let $A=\begin{bmatrix}1&2\\3&5\end{bmatrix}$ and $O=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

then

$$A+O=\begin{bmatrix}1&2\\3&5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}1&2\\3&5\end{bmatrix}=A$$

$$O+A=\begin{bmatrix}0&0\\0&0\end{bmatrix}+\begin{bmatrix}1&2\\3&5\end{bmatrix}=\begin{bmatrix}1&2\\3&5\end{bmatrix}=A$$

Additive Inverse of a Matrix

If A and B are two matrices of same order such that $A + B = O = B + A$ then A and B are called additive inverse of each other.

Additive inverse of any matrix A is obtained by changing the signs of all the non zero entries of A.

Let $A=\begin{bmatrix}1&2&1\\0&-1&-2\\3&1&0\end{bmatrix}$

then

$$B=(-A)=-\begin{bmatrix}1&2&1\\0&-1&-2\\3&1&0\end{bmatrix}=\begin{bmatrix}-1&-2&-1\\0&1&2\\-3&-1&0\end{bmatrix}$$

is additive inverse of A. It can be verified as:

$$A+B=\begin{bmatrix}1&2&1\\0&-1&-2\\3&1&0\end{bmatrix}+\begin{bmatrix}-1&-2&-1\\0&1&2\\-3&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}(1)+(-1) & (2)+(-2) & (1)+(-1)\\ 0+0 & (-1)+(1) & (-2)+(2)\\ (3)+(-3) & (1)+(-1) & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

$$B+A=\begin{bmatrix}-1&-2&-1\\0&1&2\\-3&-1&0\end{bmatrix}+\begin{bmatrix}1&2&1\\0&-1&-2\\3&1&0\end{bmatrix}$$

$$=\begin{bmatrix}(-1)+(1) & (-2)+(2) & (-1)+(1)\\ 0+0 & (1)+(-1) & (2)+(-2)\\ (-3)+(3) & (-1)+(1) & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Since A + B = O = B + A

Therefore B is additive inverse of A.

Exercise 1.3

1. Which of the following matrices are conformable for addition?

$A=\begin{bmatrix}2&1\\-1&3\end{bmatrix}$, $B=\begin{bmatrix}3\\1\end{bmatrix}$,

$C=\begin{bmatrix}1&0\\2&-1\\1&-2\end{bmatrix}$, $D=\begin{bmatrix}2+1\\3\end{bmatrix}$,

$E=\begin{bmatrix}-1&0\\1&2\end{bmatrix}$,

$F=\begin{bmatrix}3&2\\1+1&-4\\3+2&2+1\end{bmatrix}$ **Ans. (i)**

$A=\begin{bmatrix}2&1\\-1&3\end{bmatrix}$, and $E=\begin{bmatrix}-1&0\\1&2\end{bmatrix}$

are conformable for addition.

(ii) $B=\begin{bmatrix}3\\1\end{bmatrix}$ and $D=\begin{bmatrix}2+1\\3\end{bmatrix}$

are conformable for addition.

(iii) $C=\begin{bmatrix}1&0\\2&-1\\1&-2\end{bmatrix}$ and $F=\begin{bmatrix}3&2\\1+1&-4\\3+2&2+1\end{bmatrix}$

are conformable for addition.

2. Find the additive inverse of following matrices.

$A=\begin{bmatrix}2&4\\-2&1\end{bmatrix}$,

$B=\begin{bmatrix}1&0&-1\\2&-1&3\\3&-2&1\end{bmatrix}$, $C=\begin{bmatrix}4\\-2\end{bmatrix}$

$D=\begin{bmatrix}1&0\\-3&-2\\2&1\end{bmatrix}$,

$E=\begin{bmatrix}1&0\\0&1\end{bmatrix}$,

$F=\begin{bmatrix}\sqrt{3}&1\\-1&\sqrt{3}\end{bmatrix}$

Ans.

(i) $A=\begin{bmatrix}2&4\\-2&1\end{bmatrix}$

Additive inverse of Matrix A is

$$-A=-\begin{bmatrix}2&4\\-2&1\end{bmatrix}\Rightarrow -A=\begin{bmatrix}-2&-4\\2&-1\end{bmatrix}$$

(ii) $B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Additive inverse of Matrix B is

$$-B = -\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

(iii) $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Additive inverse of Matrix C is

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} \Rightarrow -C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

(iv) $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

Additive inverse of Matrix D is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} \Rightarrow -D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(v) $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Additive inverse of Matrix E is

$$-E = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow -E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(vi) $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Additive inverse of Matrix F is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow -F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

3. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, **then find,**

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (v) 2A

(vi) (–1)B (vii) (–2)C

(viii) 3D (ix) 3C

Ans. (i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1+1 & 1+2 \\ 2+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix} =$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-2 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1+1 & 2+3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+0 & 2+1 & 0+3 \\ -1+2 & 0+0 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(v) $2A = 2\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) $-1(B) = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) $(-2)C = (-2)\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$$

(viii) $3D = 3\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 6 & 0 & 3 \end{bmatrix}$

(ix) $3C = 3\begin{bmatrix}1 & -1 & 2\end{bmatrix} = \begin{bmatrix}3 & -3 & 6\end{bmatrix}$

4. Perform the indicated operations and simplify the following.

(i) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

(iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}$
$+(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix})$

(iv) $\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$

(vi) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Ans. (i) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+0+1 & 0+2+1\\0+3+1 & 1+0+0\end{bmatrix} = \begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+0-1 & 0+2-1\\0+3-1 & 1+0-0\end{bmatrix}$

$= \begin{bmatrix}0 & 1\\2 & 1\end{bmatrix}$

(iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}+(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix})$

$= \begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}1-2 & 0-2 & 2-2\end{bmatrix}$

$= \begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\end{bmatrix}$

$= \begin{bmatrix}2-1 & 3-2 & 1+0\end{bmatrix}$

$= \begin{bmatrix}1 & 1 & 1\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+1 & 3+1\\-1+2 & -1+2 & -1+2\\0+3 & 1+3 & 2+3\end{bmatrix}$

$= \begin{bmatrix}2 & 3 & 4\\1 & 1 & 1\\3 & 4 & 5\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+0 & 3-2\\2-2 & 3-1 & 1-0\\3+0 & 1+2 & 2-1\end{bmatrix}$

$= \begin{bmatrix}2 & 2 & 1\\0 & 2 & 1\\3 & 3 & 1\end{bmatrix}$

(vi) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

$= \begin{bmatrix}1+2+1 & 2+1+1\\0+1+1 & 1+0+1\end{bmatrix}$

$= \begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$

5. For the matrices

$A = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix},$

$B = \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and**

$C=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$ **verify the following rules.**

(i) $A+C=C+A$

(ii) $A+B=B+A$

(iii) $B+C=C+B$

(iv) $A+(B+A)=2A+B$

(v) $(C-B)+A=C+(A-B)$

(vi) $2A+B=A+(A+B)$

(vii) $(C-B)-A=(C-A)-B$

(viii) $(A+B)+C=A+(B+C)$

(ix) $A(B-C)=(A-C)+B$

(x) $2A+2B=2(A+B)$

Ans.

(i) $A+C=C+A$

L.H.S = A + C

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\ 2+0 & 3-2 & 1+3\\ 1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\ 2 & 1 & 4\\ 2 & 0 & 2\end{bmatrix}$$

R.H.S = C + A

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\ 0+2 & -2+3 & 3+1\\ 1+1 & 1-1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\ 2 & 1 & 4\\ 2 & 0 & 2\end{bmatrix}$$

L.H.S = R.H.S

(ii) $A+B=B+A$

L.H.S = A + B

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

R.H.S = B + A

$$=\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2 & 1+3\\ 2+2 & -2+3 & 2+1\\ 3+1 & 1-1 & 3+0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

L.H.S. = R.H.S

(iii) $B+C=C+B$

L.H.S = B + C

$$=\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 1+0\\ 2+0 & -2-2 & 2+3\\ 3+1 & 1+1 & 3+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

R.H.S = C + B

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0-1 & 0+1\\ 0+2 & -2-2 & 3+2\\ 1+3 & 1+1 & 3+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

L.H.S = R.H.S.

(iv) $A+(B+A)=2A+B$

L.H.S = A+ (B+A)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S = 2A + B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 4-1 & 6+1\\ 4+2 & 6-2 & 2+2\\ 2+3 & -2+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

L.H.S = R.H.S

(v) $(C-B)+A=C+(A-B)$

L.H.S. = (C–B) + A

$$C-B=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-1 & 0+1 & 0-1\\ 0-2 & -2+2 & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}$$

$$(C-B)+A=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2+1 & 1+2 & -1+3\\ -2+2 & 0+3 & 1+1\\ -2+1 & 0-1 & -1+0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

R.H.S. = C + (A – B)

$$A-B=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+1 & 3-1\\ 2-2 & 3+2 & 1-2\\ 1-3 & -1-1 & 0-3\end{bmatrix}$$

$$=\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$C+(A-B)=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}-1+0 & 0+3 & 0+2\\ 0+0 & -2+5 & 3-1\\ 1-2 & 1-2 & 2-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

L.H.S = R.H.S.

(vi) $2A+B=A+(A+B)$

L.H.S = 2A + B

$$2A+B=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 4-1 & 6+1\\ 4+2 & 6-2 & 2+2\\ 2+3 & -2+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S. = A+(A+B)

$$A+(A+B)=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+$$

$$\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 3 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

L.H.S. = R.H.S.

(vii) $(C-B)-A=(C-A)-B$

L.H.S. = (C – B) – A

$$C-B=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-1 & 0+1 & 0-1\\ 0-2 & -2+2 & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}$$

$$(C-B)-A=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2-1 & 1-2 & -1-3\\ -2-2 & 0-3 & 1-1\\ -2-1 & 0+1 & -1-0\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix}$$

R.H.S. = (C – A) – B

$$(C-A)=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1-1 & 0-2 & 0-3\\ 0-2 & -2-3 & 3-1\\ 1-1 & 1+1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -2 & -3\\ -2 & -5 & 2\\ 0 & 2 & 2\end{bmatrix}$$

$$(C-A)-B=\begin{bmatrix}-2 & -2 & -3\\ -2 & -5 & 2\\ 0 & 2 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-2-1 & -2+1 & -3-1\\ -2-2 & -5+2 & 2-2\\ 0-3 & 2-1 & 2-3\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix}$$

L.H.S = R.H.S.

(viii) (A+B) + C = A + (B + C)

L.H.S = (A+B) + C

$$A+B=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

R.H.S = A + (B + C)

$$B+C=\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 1+0\\ 2+0 & -2-2 & 2+3\\ 3+1 & 1+1 & 2+3\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 2-1 & 3+1\\ 2+2 & 3-4 & 1+5\\ 1+4 & -1+5 & 0+5\end{bmatrix}=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

R.H.S = R.H.S

(ix) A + (B − C) = (A − C) + B

L.H.S = A + (B − C)

$$B-C=\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1-0 & 1-0\\ 2-0 & -2+2 & 2-3\\ 3-1 & 1-1 & 3-2\end{bmatrix}=\begin{bmatrix}2 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2-1 & 3+1\\ 2+2 & 3+0 & 1-1\\ 1+2 & -1+0 & 0+1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

R.H.S = (A−C)+B

$$A-C=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-0 & 3-0\\ 2-0 & 3+2 & 1-3\\ 1-1 & -1-1 & 0-2\end{bmatrix}=\begin{bmatrix}2 & 2 & 3\\ 2 & 5 & -2\\ 0 & -2 & -2\end{bmatrix}$$

$$(A-C)+B=\begin{bmatrix}2 & 2 & 3\\ 2 & 5 & -2\\ 0 & -2 & -2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 2-1 & 3+1\\ 2+2 & 5-2 & -2+2\\ 0+3 & -2+1 & -2+3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

L.H.S. = R.H.S.

(x) 2A + 2B = 2(A + B)

L.H.S. = **2A+2B**

$$2A+2B = 2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + 2\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix} + \begin{bmatrix}2 & -2 & 2\\ 4 & -4 & 4\\ 6 & 2 & 6\end{bmatrix}$$

$$= \begin{bmatrix}2+2 & 4-2 & 6+2\\ 4+4 & 6-4 & 2+4\\ 2+6 & -2+2 & 0+6\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

R.H.S= **2 (A+B)**

$$2(A+B) = 2\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$= 2\left(\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}\right)$$

$$= 2\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

L.H.S = R.H.S

6. If $A = \begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ **and** $B = \begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$,

find (i) **3A–2B (ii) $2A^t - 3B^t$.**

Ans. (i)

$$3A - 2B = 3\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix} - 2\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$$

$$= \begin{bmatrix}3 & -6\\ 9 & 12\end{bmatrix} - \begin{bmatrix}0 & 14\\ -6 & 16\end{bmatrix}$$

$$= \begin{bmatrix}3-0 & -6-14\\ 9+6 & 12-16\end{bmatrix}$$

$$= \begin{bmatrix}3 & -20\\ 15 & -4\end{bmatrix}$$

(ii) $\mathbf{2A^t - 3B^t}$

$$A = \begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$$

$$A^t = \begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}$$

$$2A^t = 2\begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}$$

$$= \begin{bmatrix}2 & 6\\ -4 & 8\end{bmatrix}$$

$$B = \begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$$

$$B^t = \begin{bmatrix}0 & -3\\ 7 & 8\end{bmatrix}$$

$$3B^t = 3\begin{bmatrix}0 & -3\\ 7 & 8\end{bmatrix}$$

$$= \begin{bmatrix}0 & -9\\ 21 & 24\end{bmatrix}$$

$$2A^t - 3B^t = \begin{bmatrix}2 & 6\\ -4 & 8\end{bmatrix} - \begin{bmatrix}0 & -9\\ 21 & 24\end{bmatrix}$$

$$= \begin{bmatrix}2-0 & 6+9\\ -4-21 & 8-24\end{bmatrix}$$

$$= \begin{bmatrix}2 & 15\\ -25 & -16\end{bmatrix}$$

7. If $2\begin{bmatrix}2 & 4\\ -3 & a\end{bmatrix} + 3\begin{bmatrix}1 & b\\ 8 & -4\end{bmatrix} = \begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$, **then find a and b.**

Ans. $2\begin{bmatrix}2 & 4\\ -3 & a\end{bmatrix} + 3\begin{bmatrix}1 & b\\ 8 & -4\end{bmatrix} = \begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$

$$\begin{bmatrix}4 & 8\\ -6 & 2a\end{bmatrix} + \begin{bmatrix}3 & 3b\\ 24 & -12\end{bmatrix} = \begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$\Rightarrow$ $8+3b = 10$ .... (i)

$2a - 12 = 1$ .... (ii)

From (i)

$3b = 10 - 8$

$3b = 2$

$b = \frac{2}{3}$

From (ii)

$2a = 1 + 12$

$a = \frac{13}{2}$

8. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$,

then verify that

(i) $(A+B)^t = A^t + B^t$

(ii) $(A-B)^t = A^t - B^t$

(iii) $A + A^t$ is symmetric

(iv) $A - A^t$ is skew symmetric

(v) $B + B^t$ is symmetric

(vi) $B - B^t$ is skew symmetric

Ans. (i) $(A+B)^t = A^t + B^t$

L.H.S $= (A+B)^t$

$$(A+B) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 \\ 0+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

R.H.S $= A^t + B^t$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+2 \\ 2+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

L.H.S. = R.H.S.

(ii) $(A-B)^t = A^t - B^t$

L.H.S. $= (A-B)^t$

$$(A - B) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} 1-1 & 2-1 \\ 0-2 & 1-0 \end{bmatrix}$$

$$(A-B) = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$(A-B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

R.H.S $= A^t - B^t$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

L.H.S = R.H.S

(iii) $A + A^t$ **is symmetric**

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = A + A^t$$

So, $A + A^t$ is symmetric.

(iv) $A - A^t$ **is skew symmetric**

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$(A - A^t)^t = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$= -(A - A^t)^t$ is skew symmetric

(v) **$B+B^t$ is symmetric**

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$\left(B + B^t\right)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$= (B+B^t)$ is symmetric

(vi) $B-B^t$ is skew symmetric

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\left(B - B^t\right)^t = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$= -\left(B - B^t\right)$ is skew symmetric

Multiplication of Matrices.

Two matrices A and B are conformable for multiplication, giving product AB if the number of columns of A is equal to the number of rows of B.

e.g., let $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Here number of columns of A is equal to the number of rows of B. So A and B matrices are conformable for multiplication.

Examples

(i) **If** $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$,

then $AB = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times2+2\times3 & 1\times0+2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & 0+2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \end{bmatrix}$$

It is a matrix of order 1-by-2.

(ii)

If $A = \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$, then

$$AB=\begin{bmatrix}1 & 3\\ 2 & -3\end{bmatrix}\times\begin{bmatrix}-1 & 0\\ 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+3\times3 & 1\times0+3\times2\\ 2(-1)+(-3)(3) & 2\times0+(-3)(2)\end{bmatrix}$$

$$=\begin{bmatrix}-1+9 & 0+6\\ -2-9 & 0-6\end{bmatrix}=\begin{bmatrix}8 & 6\\ -11 & -6\end{bmatrix},$$ is a 2-by-2 matrix.

Associative Law under Multiplication

If A, B and C are three matrices conformable for multiplication then associative law under multiplication is given as

$$(AB)C=A(BC)$$

e.g., If $A=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}$ and

$C=\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$, then

L.H.S. = (AB)C

$$=\left(\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}\right)\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2\times0+3\times3 & 2\times1+3\times1\\ -1\times0+0\times3 & -1\times1+0\times1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0+9 & 2+3\\ 0+0 & -1+0\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}9 & 5\\ 0 & -1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}9\times2+5\times(-1) & 9\times2+5\times0\\ 0\times2+(-1)\times(-1) & 0\times2+(-1)\times0\end{bmatrix}$$

$$=\begin{bmatrix}18-5 & 18+0\\ 0+1 & 0+0\end{bmatrix}=\begin{bmatrix}13 & 18\\ 1 & 0\end{bmatrix}$$

$$R.H.S=A(BC)=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0\times2+1\times(-1) & 0\times2+1\times0\\ 3\times2+1\times(-1) & 3\times2+1\times0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2(-1)+3\times5 & 2\times0+3\times6\\ (-1)(-1)+0\times5 & -1\times0+0\times6\end{bmatrix}$$

$$=\begin{bmatrix}-2+15 & 0+18\\ 1+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}13 & 18\\ 1 & 0\end{bmatrix}=(AB)C$$

The associative law under multiplication of matrices is verified.

Distributive Laws of Multiplication over Addition and Subtraction

(a) Let A, B and C be three matrices. Then distributive laws of multiplication over addition are given below.

(i) $A(B+C)=AB+AC$

(Left distributive law)

(ii) $(A+B)C=AC+BC$

(Right distributive law)

Let $A=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}$

and $C=\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$ then in (i)

L.H.S. = A(B+C)

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}+\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0+2 & 1+2\\ 3-1 & 1+0\end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times2 & 2\times3+3\times1 \\ -1\times2+0\times2 & -1\times3+0\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+6 & 6+3 \\ -2+0 & -3+0 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ -2 & -3 \end{bmatrix}$$

R.H.S. = AB + AC

$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times0+3\times3 & 2\times1+3\times1 \\ -1\times0+0\times3 & -1\times1+0\times1 \end{bmatrix} + \begin{bmatrix} 2\times2+3\times(-1) & 2\times2+3\times0 \\ -1\times2+0\times(-1) & -1\times2+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 5 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+1 & 5+4 \\ 0-2 & -1-2 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ -2 & -3 \end{bmatrix} = \text{L.H.S}$$

Which shows that

$$A(B+C) = AB + AC;$$

b) **Let** $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

and $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then in (i)

L.H.S. = A(B − C)

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -1-2 & 1-1 \\ 1-1 & 0-2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(-3)+(3)(0) & 2(0)+3(-2) \\ (0)(-3)+1\times0 & 0\times0+(1)(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -6+0 & 0-6 \\ 0+0 & 0-2 \end{bmatrix} = \begin{bmatrix} -6 & -6 \\ 0 & -2 \end{bmatrix}$$

R.H.S. = AB − AC

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-1)+3(1) & 2(1)+3(0) \\ 0(-1)+1(1) & 0(1)+1(0) \end{bmatrix} - \begin{bmatrix} 2\times2+3\times1 & 2\times1+3\times2 \\ 0\times2+1\times1 & 0\times1+1\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 7 & 8 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1-7 & 2-8 \\ 1-1 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -6 \\ 0 & -2 \end{bmatrix}$$

Which shows that

$$A(B-C) = AB - AC$$

Commutative Law of Multiplication of Matrices

Consider the matrices $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ then

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times1+1\times0 & 0\times0+1(-2) \\ 2\times1+3\times0 & 2\times0+3(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & -6 \end{bmatrix}$$

and

$$BA = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times0+0\times2 & 1\times1+0\times3 \\ 0\times0+(-2)\times2 & 0\times1+3(-2) \end{bmatrix}$$

$$=\begin{bmatrix}0 & 1\\ -4 & -6\end{bmatrix}$$

Which shows that. $AB \neq BA$.

Note: Commutative law under multiplication in matrices does not hold in general i.e., if A and B are two matrices then $AB \neq BA$.

Commutative law under multiplication holds in particular case.

e.g., If $A=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 0\\ 0 & 4\end{bmatrix}$

then

$$AB=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}-3 & 0\\ 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2\times(-3)+0\times0 & 2\times0+0\times4\\ 0\times(-3)+1\times0 & 0\times0+1\times4\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 0\\ 0 & 4\end{bmatrix}$$

and $$BA=\begin{bmatrix}-3 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-3\times2+0\times0 & -3\times0+0\times1\\ 0\times2+4\times0 & 0\times0+4\times1\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 0\\ 0 & 4\end{bmatrix}$$

Which shows that $AB = BA$.

Multiplicative Identity of a Matrix.

Let A be a matrix. Another matrix B is called the identity matrix of A under multiplication if

$AB = A = BA$

If $A=\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then

$$AB=\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+2\times0 & 1\times0+2\times1\\ 0\times1+(-3)\times0 & 0\times0+(-3)(1)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times0 & 1\times2+0\times(-3)\\ 0\times1+1\times0 & 0\times2+1\times(-3)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}$$

Which shows that $AB = A = BA$.

Verification of $(AB)^t = B^t A^t$.

If A and B are two matrices and A^t, B^t are their respective transpose, then $(AB)^t = B^t A^t$.

e.g., $A=\begin{bmatrix}2 & 1\\ 0 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -2 & 0\end{bmatrix}$

L.H.S. $=(AB)^t$

$$=\left(\begin{bmatrix}2 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 3\\ -2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2\times1+1\times(-2) & 2\times3+1\times0\\ 0\times1+(-1)\times(-2) & 0\times3+(-1)\times0\end{bmatrix}^t$$

$$=\begin{bmatrix}2-2 & 6+0\\ 0+2 & 0+0\end{bmatrix}^t=\begin{bmatrix}0 & 6\\ 2 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}0 & 2\\ 6 & 0\end{bmatrix}$$

R.H.S. $= B^t A^t$,

$$(A)^t = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}^t = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$(B)^t = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}^t = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+(-2)\times1 & 1\times0+(-2)(-1) \\ 3\times2+0\times1 & 3\times0+0\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 0+2 \\ 6+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 6 & 0 \end{bmatrix} = \text{L.H.S}$$

L.H.S = R.H.S

Thus $(AB)^t = B^t A^t$.

Exercise 1.4

1. Which of the following product of matrices is conformable for multiplication?

Ans. (i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Number of Columns = Number of Rows

∴ product is possible.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Number of columns = Number of Rows.

∴ product is possible.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Number of columns ≠ Number of Rows.

∴ product is not possible.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Number of columns = Number of Rows.

∴ product is possible.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Number of Columns = Number of Rows.

∴ Product is possible.

2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible).

(i) $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$= \begin{bmatrix} 3(6) + 0(5) \\ -1(6)+2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) $BA = \begin{bmatrix} 6 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

∴ Product is not possible.

Because number of columns ≠ number of rows.

3. Find the following products.

Ans. (i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}1(4)+2(0)\end{bmatrix}$

$=\begin{bmatrix}4+0\end{bmatrix}$

$=\begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

$=\begin{bmatrix}1(5)+2(-4)\end{bmatrix}$

$=\begin{bmatrix}5-8\end{bmatrix}$

$=\begin{bmatrix}-3\end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}-3(4)+0(0)\end{bmatrix}=\begin{bmatrix}-12\end{bmatrix}$

(iv) $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=\begin{bmatrix}6(4)+(0)(0)\end{bmatrix}=\begin{bmatrix}24\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$=\begin{bmatrix}1(4)+2(0) & 1(5)+2(-4)\\-3(4)+0(0) & -3(5)+0(-4)\\6(4)+-1(0) & 6(5)+(-1)(-4)\end{bmatrix}$

$=\begin{bmatrix}4+0 & 5-8\\-12+0 & -15+0\\24+0 & 30+4\end{bmatrix}$

$=\begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

4. Multiply the following matric

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{4}\\4 & 4\end{bmatrix}$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Ans. (a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$=\begin{bmatrix}2(2)+3(3) & 2(-1)+3(0)\\1(2)+1(3) & 1(-1)+1(0)\\0(2)+(-2)(3) & 0(-1)+(-2)(0)\end{bmatrix}$

$=\begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$=\begin{bmatrix}1(1)+2(3)+3(-1) & 1(2)+2(4)+3(1)\\4(1)+15(3)+6(-1) & 4(2)+5(4)+6(1)\end{bmatrix}$

$=\begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$

$$=\begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

$$=\begin{bmatrix}1(1)+2(4) & 1(2)+2(5) & 1(3)+2(6)\\3(1)+4(4) & 3(2)+4(5) & 3(3)+4(6)\\-1(1)+1(4) & -1(2)+1(5) & -1(3)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

$$=\begin{bmatrix}8(2)+5(-4) & 8\left(\frac{-5}{2}\right)+5(4)\\6(2)+4(-4) & 6\left(\frac{-5}{2}\right)+4(4)\end{bmatrix}$$

$$=\begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}=\begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$$=\begin{bmatrix}-1(0)+2(0) & -1(0)+2(0)\\1(0)+3(0) & 1(0)+3(0)\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

5. Let $A=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$. **Verify whether**

(i) AB = BA.

(ii) A(BC) = (AB)C

(iii) A(B+C)=AB+AC

(iv) A(B−C)=AB−AC

Ans. (i) AB = BA.

To check whether AB = BA Or not

$$AB=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\2-0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(-1)+2(2) & 1(3)+2(0)\\-3(-1)+-5(2) & -3(3)+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\3-10 & -9+0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\-7 & -9\end{bmatrix}$$

So **AB ≠ BA**

(ii) A(BC) = (AB)C

L.H.S = A(BC)

$$BC=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1(2)+2(1) & 1(1)+2(3)\\-3(2)+-5(1) & -3(1)+-5(3)\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 1+6\\-6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}-1(4)+3(-11) & -1(7)+3(-18)\\2(4)+0(-11) & 2(7)+0(-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\8+0 & 14+0\end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

R.H.S = (AB)C

$$(AB) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10(2)+(-17)(1) & -10(1)+(-17)(3) \\ 2(2)+4(1) & 2(1)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

Hence A(BC) = (AB)C

(iii) A (B + C) = AB + AC

L.H.S = A (B + C)

$$(B + C) = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

L.H.S.

AB + AC

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1(2)+3(1) & -1(1)+3(3) \\ 2(2)+0(1) & 2(1)+0(3) \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

6. For the matrices.

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}, C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

Verify that(i) $(AB)^t = B^t A^t$ **(ii)** $(BC)^t = C^t B^t$.

Ans. (i) $(AB)^t = B^t A^t$

L.H.S = $(AB)^t$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+3(-3) & -1(2)+3(-5) \\ 2(1)+0(-3) & 2(2)+0(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

R.H.S = $B^t A^t$

$$A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+(-3)(3) & 1(2)+(-3)(0) \\ 2(-1)+(-5)(3) & 2(2)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & 2-0 \\ -2-15 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

L.H.S = R.H.S

Hence $(AB)^t = B^t A^t$

(ii) $(BC)^t = C^t B^t$

L.H.S $= (BC)^t$

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-2)+2(3) & 1(6)+2(-9) \\ -3(-2)+-5(3) & -3(6)+-5(-9) \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}$$

$$(BC)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

R.H.S $= C^t B^t$

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$C^t B^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -2(1)+3(2) & -2(-3)+3(-5) \\ 6(1)+2(-9) & 6(-3)+-9(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

L.H.S = R.H.S

Hence $(BC)^t = C^t B^t$

Determinant of a 2-by-2 Matrix.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by-2 square matrix. The determinant of A, denoted by det A or |A| is defined as $|A| = \det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = \lambda \in R$ e.g.,

Let $B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$. Then $|B| = \det B = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}$

$= 1\times3-(-2)(1) = 3+2 = 5$

If $M = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, then

$$\det M = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 2\times3 - 1\times6 = 0$$

Singular and non-singular matrix.

A square matrix A is called singular if determinant of A is equal to zero.

i.e., |A|=0.

For example, $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ is a singular matrix, since $\det A = 1\times0 - 0\times2 = 0$

A square matrix A is called non-singular if the determinant of A is not equal to zero.

i.e., $|A| \neq 0$

For example $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ is non-singular, since $\det A = 1\times2 - 0\times1 = 2 \neq 0$.

Note that, each square matrix with real entries is either singular or non-singular.

Adjoint of a Matrix.

Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as Adj A.

i.e., $\text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

e.g., if $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$, then

$$\text{Adj } A = \begin{bmatrix} 0 & -2 \\ -3 & 1 \end{bmatrix}$$

If $B = \begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix}$, then $\text{Adj } B = \begin{bmatrix} -4 & 1 \\ -3 & 2 \end{bmatrix}$

Multiplicative inverse of a non-singular matrix.

Let A and B be two non-singular square matrices of same order. Then A and B are said to be multiplicative inverse of each other if

$$AB = BA = I$$

The inverse of A is denoted by A^{-1}, thus $AA^{-1} = A^{-1}A = I$.

Inverse of a matrix is possible only if matrix is non-singular.

Inverse of a Matrix using Adjoint

Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix. To find the inverse of M, i.e., M^{-1}, first we find the determinant as inverse is possible only of a non-singular matrix.

$$|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$$

and $\text{Adj } M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, then

$$M^{-1} = \frac{\text{Adj } M}{|M|}$$

e.g., Let $A = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}$.

Then $|A| = -6-(-1) = 6+1 = -5 \neq 0$

$|A| = -6-(-1) = -6+1 = -5 \neq 0.$

Thus $A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{\begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}}{-5}$

$$= \frac{-1}{5}\begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-2}{5} \end{bmatrix}$$

and $AA^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-2}{5} \end{bmatrix}$

$$= \begin{bmatrix} \frac{6}{5}-\frac{1}{5} & \frac{2}{5}-\frac{2}{5} \\ -\frac{3}{5}+\frac{3}{5} & -\frac{1}{5}+\frac{6}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = A^{-1}A$$

Verification of $(AB)^{-1} = B^{-1}A^{-1}$

Let $A = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$

Then $\det A = 3 \times 0 - (-1) \times 1 = 1 \neq 0$

And $\det B = 0 \times 2 - 3(-1) = 3 \neq 0$

Therefore, A and B are invertible i.e., their inverses exist.

Then, to verify the law of inverse of the product, take

AB

$$= \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times0+1\times3 & 3\times(-1)+1\times2 \\ -1\times0+0\times3 & -1\times(-1)+0\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \det(AB) = \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} = 3 \neq 0$$

and L.H.S. = $(AB)^{-1}$

$$(AB)^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{bmatrix}$$

R.H.S.= $B^{-1}A^{-1}$, where $B^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$,

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\frac{1}{1}\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2\times0+1\times1 & 2\times(-1)+1\times3 \\ -3\times0+0\times1 & -3\times(-1)+0\times3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 0+1 & -2+3 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{bmatrix} = (AB)^{-1}$$

Thus the law $(AB)^{-1} = B^{-1}A^{-1}$ is verified.

Exercise 1.5

1. Find the determinant of the following matrices.

Ans. (i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= -1(0) - 2(1)$$

$$= 0 - 2 = -2$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

$$|B| = 1(-2) - 2(3)$$

$$= -2 - 6$$

$$= -8$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$$|C| = 3(2) - 3(2)$$

$$= 6 - 6 = 0$$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$|D| = 3(4) - 1(2)$$

$$= 12 - 2 = 10$$

2. Find which of the following matrices are singular or non-singular?

Ans. (i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$$

$$= 3(4) - 2(6)$$

$$= 12 - 12$$

$= 0$ singular

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

$$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$$

$= 4(2) - 3(1) = 8 - 3 = 5$ non-singular

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$$

$$= 7(5) - 3(-9)$$

$$= 35 + 27$$

$= 62 \neq 0$ non-singular

(iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$

$= 5(4) - (-2)(-10)$

$= 20 - 20$

$= 0$ singular

3. Find the multiplicative inverse (if it exists) of each.

Ans. (i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$

$= -1(0) - 2(3)$

$= -6$

$\text{Adj}A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$= \frac{1}{-6}\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$

$= 1(-5) - (-3)(2)$

$= -5 + 6$

$= 1 \neq 0$

$\text{Adj } B = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \text{adj} B$

$= \frac{1}{1}\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

(iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$

$= -2(-9) - 3(6)$

$= 18 - 18 = 0$

C^{-1} does not exist.

(iv) $D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$

$= \frac{1}{2}(2) - 1\left(\frac{3}{4}\right)$

$= 1 - \frac{3}{4}$

$= \frac{4-3}{4} = \frac{1}{4} \neq 0$

$\text{Adj } D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

$D^{-1} = \frac{1}{|D|} \text{adj.} D$

$= \frac{1}{1/4}\begin{bmatrix} 2 & -3/4 \\ -1 & 1/2 \end{bmatrix}$

$= 4\begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$

4. If $A=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$, then

(i) $A(\text{Adj } A) = (\text{Adj } A)\,A = (\det A)I$

(ii) $BB^{-1} = I = B^{-1}B$

Ans. (i) A(Adj A)=(Adj A) A = (det A)I

$$A=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$$

$$\text{Adj}\,A=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$A(\text{Adj}A)=\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1(6)+2(-4) & 1(-2)+2(1)\\4(6)+6(-4) & 4(-2)+6(1)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & -2+2\\24-24 & -8+6\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}$$

Now $(\text{Adj}A)A=\begin{bmatrix}6 & -2\\-4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 6\end{bmatrix}$

$$=\begin{bmatrix}6(1)+-2(4) & 6(2)+-2(6)\\-4(1)+1(4) & -4(2)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\-4+4 & -8+6\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}$$

Also (det A)I

$$\det A=\begin{vmatrix}1 & 2\\4 & 6\end{vmatrix}$$

$$=1(6)-2(4)=6-8=-2$$

$$(\det A)I=-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 0\\0 & -2\end{bmatrix}$$

Hence: $A(\text{Adj}A) = (\text{Adj}A)\,A = (\det A)I$

(ii) $B=\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$

$$|B|=\begin{vmatrix}3 & -1\\2 & -2\end{vmatrix}=3(2)-2(-1)$$

$$=-6+2=-4\neq 0$$

$$\text{Adj}B=\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\text{Adj}\,B$$

$$=\frac{1}{-4}\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}=\frac{1}{4}\begin{bmatrix}2 & -1\\2 & -3\end{bmatrix}$$

$$BB^{-1}=\frac{1}{4}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\2 & -3\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}3(2)+(-1)(2) & 3(-1)+(-1)(-3)\\2(2)+(-2)(2) & 2(-1)+(-2)(-3)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}6-2 & -3+3\\4-4 & -2+6\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

Similarly:

$$B^{-1}B=\frac{1}{4}\begin{bmatrix}2 & -1\\2 & -3\end{bmatrix}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}2(3)+(-1)(2) & 2(-1)+(-1)(-2)\\2(3)+(-3)(2) & 2(-1)+(-3)(-2)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

Hence: $BB^{-1} = I = B^{-1}B$

5. Determine whether the given matrices are multiplicative inverses of each other.

Ans. (i) $\begin{bmatrix}3 & 5\\4 & 7\end{bmatrix}$ and $\begin{bmatrix}7 & -5\\-4 & 3\end{bmatrix}$

$$\begin{bmatrix}3 & 5\\ 4 & 7\end{bmatrix}\begin{bmatrix}7 & -5\\ -4 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3(7)+5(-4) & 3(-5)+5(3)\\ 4(7)+7(-4) & 4(-5(+7(3)\end{bmatrix}$$

$$=\begin{bmatrix}21-20 & -15+15\\ 28-28 & -20+21\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

∴ Given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ and $\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$

$$\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}-3 & 2\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}1(-3)+2(2) & 1(2)+2(-1)\\ 2(-3)+3(2) & 2(2)+3(-1)\end{bmatrix}$$

$$=\begin{bmatrix}-3+4 & 2-2\\ -6+6 & 4-3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

6. If $A=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}$, $B=\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$, $D=\begin{bmatrix}3 & 1\\ -2 & 2\end{bmatrix}$**, then verify that**

(i) $(AB)^{-1}=B^{-1}A^{-1}$

(ii) $(DA)^{-1}=A^{-1}D^{-1}$

Ans. (i) $\mathbf{(AB)^{-1}=B^{-1}A^{-1}}$

L.H.S = $(AB)^{-1}$

$$AB=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}4(-4)+0(1) & 4(-2)+0(-1)\\ -1(-4)+2(1) & -1(-2)+2(-1)\end{bmatrix}$$

$$=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$$

$$|AB|=\begin{vmatrix}-16 & -8\\ 6 & 0\end{vmatrix}$$

$$=-16(0)-6(-8)$$

$$=0+48=48\neq 0$$

$$Adj(AB)=\begin{vmatrix}0 & 8\\ -6 & -16\end{vmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}Adj(AB)$$

$$=\frac{1}{48}\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}=\begin{bmatrix}0 & \frac{8}{48}\\ \frac{-6}{48} & \frac{-16}{48}\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{6}\\ \frac{-1}{8} & -\frac{1}{3}\end{bmatrix}$$

R.H.S = $B^{-1}A^{-1}$

$$B=\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$$

$$|B|=-4(-1)-1)(-2)=4+2=6$$

$$B^{-1}=\frac{1}{|B|}AdjB=\frac{1}{6}\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}$$

$$A=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 0\\ -1 & 2\end{vmatrix}=4(2)-(-1)(0)=8$$

$$A^{-1}=\frac{1}{|A|}Adj\,A=\frac{1}{8}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{6}\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}\frac{1}{8}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix}-1(2)+2(1) & -1(0)+2(4)\\ -1(2)+-4(1) & -1(0)+-4(4)\end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}=\begin{bmatrix}0 & \frac{8}{48}\\ -\frac{6}{48} & -\frac{16}{48}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{6}\\ -\frac{1}{8} & -\frac{1}{3}\end{bmatrix}$$

L.H.S = R.H.S

Hence: $(AB)^{-1}=B^{-1}A^{-1}$

(ii) $(DA)^{-1} = A^{-1}D^{-1}$

L.H.S $= (DA)^{-1}$

$$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(4)+1(-1) & -2(0)+1(2) \\ -2(4)+2(-1) & -2(0)+2(2) \end{bmatrix}_1$$

$$= \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$$

$$= 11(4)-(-10)(2)$$

$$= 44+20$$

$$= 64$$

$$Adj(DA) = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$(DA)^{-1} = \frac{1}{DA}Adj(DA)$$

$$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

R.H.S $= A^{-1}D^{-1}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$= 4(2)-(-1)(0)$$

$$= 8 \neq 0$$

$$A^{-1} = \frac{1}{|A|}AdjA$$

$$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|D| = 3(2)-(-2)(1)$$

$$= 6+2 = 8$$

$$D^{-1} = \frac{1}{|D|}AdjD$$

$$= \frac{1}{8}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1}D^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \cdot \frac{1}{8}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1}D^{-1} = \frac{1}{64}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 2(2)+0(2) & 2(-1)+0(3) \\ 1(2)+4(2) & 1(-1)+4(3) \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 4+0 & -2+0 \\ 2+8 & -1+12 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

L.H.S = R.H.S

Hence: $(DA)^{-1} = A^{-1}D^{-1}$

Solution of Simultaneous Linear Equations

System of two linear equations in two variables in general form is given as

$$ax+by=m$$

$$cx + dy = n$$

Where a, b, c, d, m and n are real numbers.

This system is also called simultaneous linear equations.

We discuss here the following methods of solution.

(i) **Matrix** inversion method.

(ii) **Cramer's** rule

(i) Matrix Inversion Method

Consider the system of linear questions

$$ax + by = m$$
$$cx + dy = n$$

Then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$

or $AX = B$

Where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} m \\ n \end{bmatrix}$

or $X = A^{-1} B$

$|A| = ad - bc$

or $X = \frac{\text{Adj } A}{|A|} \times B$

$\because \quad A^{-1} = \frac{\text{Adj} A}{|A|}$ and $|A| \neq 0$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{ad - bc}$

$= \begin{bmatrix} \frac{dm - bn}{ad - bc} \\ \frac{-cm + an}{ad - bc} \end{bmatrix}$

$\Rightarrow \quad x = \frac{dm - bn}{ad - bc}$ and $y = \frac{an - cm}{ad - bc}$

(ii) Cramer's Rule.

Consider the following system of linear equations.

$$ax + by = m$$
$$cx + dy = n$$

We know that

$AX = B$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and $B = \begin{bmatrix} m \\ n \end{bmatrix}$

or $\quad X = A^{-1} B$

or $\quad X = \frac{\text{Adj } A}{|A|} \times B$

or $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{|A|}$

$= \frac{\begin{bmatrix} dm - bn \\ -cm + an \end{bmatrix}}{|A|}$

$= \begin{bmatrix} \frac{dm - bn}{|A|} \\ \frac{-cm + an}{|A|} \end{bmatrix}$

or $\quad x = \frac{dm - bn}{|A|} = \frac{|A_x|}{|A|}$

and $\quad y = \frac{an - cm}{|A|} = \frac{|A_y|}{|A|}$

where $|A_x| = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$ and

$|A_y| = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$

Example 1

Solve the following system by using matrix inversion method.

$$4x - 2y = 8$$
$$3x + y = -4$$

Solution

Step 1 $\begin{bmatrix}4 & -2\\3 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-4\end{bmatrix}$

Step 2

The coefficient matrix $M=\begin{bmatrix}4 & -2\\3 & 1\end{bmatrix}$ is non-singular, since

$\det M = 4\times 1 - 3(-2) = 4 + 6 = 10 \neq 0$. So M^{-1} is possible.

Step 3

$$\begin{bmatrix}x\\y\end{bmatrix}=M^{-1}\begin{bmatrix}8\\-4\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}1 & 2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-4\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}8-8\\-24-16\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}0\\-40\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\-4\end{bmatrix}$$

$\Rightarrow$ $x = 0$ and $y = -4$

Example 2

Solve the following system of linear equations by using Cramer's rule.

$$3x - 2y = 1$$
$$-2x + 3y = 2$$

Solution

$$3x - 2y = 1$$
$$-2x + 3y = 2$$

We have

$$A=\begin{bmatrix}3 & -2\\-2 & 3\end{bmatrix},$$

$$A_x=\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix},$$

$$A_y=\begin{bmatrix}3 & 1\\-2 & 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}3 & -2\\-2 & 3\end{vmatrix}=9-4=5\neq 0 \text{ (non-singular)}$$

$$x=\frac{|A_x|}{|A|}=\frac{\begin{vmatrix}1 & -2\\2 & 3\end{vmatrix}}{5}=\frac{3+4}{5}=\frac{7}{5}$$

$$y=\frac{|A_y|}{|A|}=\frac{\begin{vmatrix}3 & 1\\-2 & 2\end{vmatrix}}{5}=\frac{6+2}{5}=\frac{8}{5}$$

$$S.S=\left\{\left(\frac{7}{5},\frac{8}{5}\right)\right\}$$

Example 3

The length of a rectangle is 6 cm less than three times its width. The perimeter of the rectangle is 140 cm. Find the dimensions of the rectangle.

(by using matrix inversion method)

Solution

If width of the rectangle is x cm, then length of the rectangle ycm

According to first condition

$$y = 3x - 6,$$

According to 2nd condition

The perimeter $= 2x + 2y = 140$

$\Rightarrow$ $x + y = 70$ .... (i)

and $3x - y = 6$ .... (ii)

In the matrix form

$$\begin{bmatrix}1 & 1\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}70\\6\end{bmatrix}$$

$$\det\begin{bmatrix}1 & 1\\3 & -1\end{bmatrix}=\begin{vmatrix}1 & 1\\3 & -1\end{vmatrix}$$

$$= 1\times(-1) - 3\times 1 = -1 - 3 = -4 \neq 0$$

We know that:

$$X = A^{-1}B \text{ and } A^{-1}=\frac{\text{Adj } A}{|A|}$$

Hence $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 70 \\ 6 \end{bmatrix}$

$= \frac{-1}{4}\begin{bmatrix} -70-6 \\ -210+6 \end{bmatrix} = \begin{bmatrix} \frac{76}{4} \\ \frac{204}{4} \end{bmatrix} = \begin{bmatrix} 19 \\ 51 \end{bmatrix}$

Thus, by the equality of matrices, width of the rectangle x = 19 cm and the length y = 51 cm.

Exercise 1.6

1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inverse method

(ii) the Cramer's rule.

(i) $2x-2y=4$

$3x+2y=6$

Matrix inverse method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

A X = B

$X = A^{-1}B$............(i)

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$

$= 2(2)-(-2)(3)$

$= 4 + 6 = 10 \neq 0$

As $|A| \neq 0$ so solution is possible

$Adj\ A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} AdjA$

$A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

Putting the values of A^{-1} and B in equation (i)

$X = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 2(4)+2(6) \\ -3(4)+2(6) \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$

$X = \begin{bmatrix} \cancel{20}^{2} \times \frac{1}{\cancel{10}} \\ 0 \times \frac{1}{10} \end{bmatrix}$

$X = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow$ $x=2$

$y=0$

$S.S. = \{(x,y)\} = \{(2,0)\}$

$S.S. = \{(2,0)\}$

(ii) $2x+y=3$

$6x+5y=1$

In matrices form

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \end{bmatrix};B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$= 2(5) - 6(1)$

$= 10 - 6$

$|A| = 4 \neq 0$

As $|A| \neq 0$, so solution is possible

$$Adj\ A = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ A$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

Putting the value of A^{-1} & B in equation i.

$$X = A^{-1}B$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5(3) + (-1)(1) \\ -6(3) + 2(1) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{4} \\ \frac{16}{4} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$\Rightarrow \quad x = \frac{7}{2}$

$y = -4$

Solution set $S.S. = \left\{\left(\frac{7}{2}, -4\right)\right\}$

(iii) $4x + 2y = 8$

$3x - y = -1$

In matrices form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix};X = \begin{bmatrix} x \\ y \end{bmatrix};B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$= 4(-1) - 3(2)$

$= -4 - 6$

$|A| = -10 \neq 0$

As $|A| \neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj} A$$

$$A^{-1}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}$$

Putting values of A^{-1} & B in equation.

$$X=A^{-1}B$$

$$X=\frac{1}{-10}\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}\begin{bmatrix}8\\ -1\end{bmatrix}$$

$$X=\frac{1}{-10}\begin{bmatrix}-1(8)+(-2)(-1)\\ -3(8)+4(-1)\end{bmatrix}$$

$$X=\frac{1}{-10}\begin{bmatrix}-8+2\\ -24-4\end{bmatrix}$$

$$=\frac{1}{-10}\begin{bmatrix}-6\\ -28\end{bmatrix}$$

$$=\begin{bmatrix}-\cancel{6}^{3}\times\frac{1}{\cancel{-10}_{5}}\\ -\cancel{28}^{14}\times\frac{1}{\cancel{-10}_{5}}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\ \frac{14}{5}\end{bmatrix}$$

$$\Rightarrow \quad x=\frac{3}{5}$$

$$y=\frac{14}{5}$$

$$S.S=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$$

(iv) $3x-2y=-6$

$5x-2y=-10$

In matrices form

$$\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-6\\ -10\end{bmatrix}$$

Let

$$A=\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix}; X=\begin{bmatrix}x\\ y\end{bmatrix}; B=\begin{bmatrix}-6\\ -10\end{bmatrix}$$

$$AX=B$$

$$\Rightarrow \quad X=A^{-1}B$$

$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$

$$=3(-2)-(5)(2)$$

$$=-6+10$$

$$|A|=4\neq 0$$

As $|A|\neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj} A$$

$$A^{-1}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$

Putting the values of A^{-1} & B in equation i.

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}\begin{bmatrix}-6\\ -10\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2(-6)+2(-10)\\ -5(-6)+3(-10)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}12+(-20)\\ 30-30\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}-8\\ 0\end{bmatrix}$$

$$=\begin{bmatrix} \cancel{-8}^{2}\times\frac{1}{\cancel{4}} \\ 0\times\frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x=-2$

$y=0$

$S.S=\{(-2,0)\}$

(v) $3x-2y=4$

$-6x+4y=7$

In matrices form

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix};X=\begin{bmatrix} x \\ y \end{bmatrix};B=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A|=\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

$=3(4)-(6)(-2)$

$=12-12$

$=0$

As $|A|=0$, so solution is not possible

(vi) $4x+y=9$

$-3x-y=-5$

In matrices form

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix};X=\begin{bmatrix} x \\ y \end{bmatrix};B=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A|=\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$=4(-1)-(-3)(1)$

$=-4+3$

$=-1\neq 0$

As $|A|\neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times Adj\ A$$

$$=\frac{1}{-1}\times\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

Putting the values in equation (i) of A^{-1} and B

$$X=A^{-1}B$$

$$X=\frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -1(9)+(-1)(-5) \\ 3(9)+4(-5) \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -9+5 \\ 27-20 \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{-1}\times-4 \\ \frac{1}{-1}\times 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$\Rightarrow \quad x = 4$

$y = -7$

$S.S. = \{(4, -7)\}$

(vii) $2x - 2y = 4$

$-5x - 2y = -10$

In matrices form

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

A X = B

$\Rightarrow X = A^{-1} B$

Let $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

AX = B

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$= 2(-2) - (-5)(-2)$

$= -4 - 10$

$|A| = -14 \neq 0$

As $|A| \neq 0$, so solution is possible

$$\text{Adj } A = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{-14} \times \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

Putting the values of A^{-1} and B in equation

(i) $X = A^{-1} B$

$$X = \frac{1}{-14} \times \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$X = \frac{1}{-14}\begin{bmatrix} -2(4) + 2(-10) \\ 5(4) + 2(-10) \end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$$

$$= \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{-28}^{2} \times \frac{1}{\cancel{-14}} \\ 0 \times \frac{1}{-14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x = 2$

$y = 0$

$S.S. = \{(2, 0)\}$

(viii) $3x - 4y = 4$

$x + 2y = 8$

In matrices form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let

$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$AX = B$

$\Rightarrow \quad X = A^{-1} B$............$i$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$= 3(2) - (1)(-4)$

$= 6 + 4$

$|A| = 10 \neq 0$

As $|A| \neq 0$, so solution is possible

$$\text{Adj}\,A=\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj}\,A$$

$$A^{-1}=\frac{1}{10}\times\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}$$

Putting the values of A^{-1} & B in equation (i)

$$X=A^{-1}B$$

$$X=\frac{1}{10}\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}\begin{bmatrix}4\\ 8\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}2(4)+4(8)\\ -1(4)+3(8)\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}8+32\\ -4+24\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}40\\ 20\end{bmatrix}$$

$$X=\begin{bmatrix}\cancel{40}^{4}\times\frac{1}{\cancel{10}}\\ \cancel{20}^{2}\times\frac{1}{\cancel{10}}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 2\end{bmatrix}$$

$\Rightarrow$ $x=4$

$y=2$

$S.S.=\{(4,2)\}$

Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

In matrices form

$$\begin{bmatrix}2 & -2\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 6\end{bmatrix}$$

Let $A=\begin{bmatrix}2 & -2\\ 3 & 2\end{bmatrix}; X=\begin{bmatrix}x\\ y\end{bmatrix}; B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\ 3 & 2\end{vmatrix}$$

$$=2(2)-3(-2)$$

$$=4+6$$

$$|A|=10\neq 0$$

As $|A|\neq 0$, so solution is possible.

A_x; - (Determinant No. 1)

In determinant 1 we change first column to constant matrix.

$$|A_x|=\begin{vmatrix}4 & -2\\ 6 & 2\end{vmatrix}$$

$$=4(2)-6(-2)$$

$$=8+12$$

$$|A_x|=20$$

$$x=\frac{|A_x|}{|A|}=\frac{20}{10}=2$$

$x=2$

$|A_y|$ (Determinant No.2)

In determinant 2 we change 2^{nd} column to constant matrix.

$$|A_y|=\begin{vmatrix}2 & 4\\ 3 & 6\end{vmatrix}$$

$$=2(6)-3(4)$$

$$=12-12$$

$$|A_y|=0$$

$$y=\frac{|A_y|}{|A|}=\frac{0}{10}=0$$

$y=0$

S.S={(2, 0)} .ans.

(ii) $2x+y=3$

$6x+5y=1$

In matrices form

$$\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

Let

$$A=\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}3\\1\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$$

$$=2(5)-6(1)$$

$$=10-6$$

$$|A|=4\neq 0$$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}3 & 1\\1 & 5\end{vmatrix}$$

$$=3(5)-1(1)$$

$$|A_x|=15-1$$

$$|A_x|=14$$

$$x=\frac{|A_x|}{|A|}=\frac{\cancel{14}^{7}}{\cancel{4}^{2}}$$

$$x=\frac{7}{2}$$

$$|A_y|=\begin{vmatrix}2 & 3\\6 & 1\end{vmatrix}$$

$$=2(1)-6(3)$$

$$|A_y|=2-18$$

$$|A_y|=-16$$

$$y=\frac{|A_y|}{|A|}=\frac{-16}{4}=-4$$

$$y=-4$$

$$S.S=\left\{\left(\frac{7}{2},-4\right)\right\}$$

(iii) $4x+2y=8$

$3x-y=-1$

In matrices form

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-1\end{bmatrix}$$

Let

$$A=\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}8\\-1\end{bmatrix}$$

$$|A|=4(-1)-3(2)$$

$$=-4-6$$

$$|A|=-10\neq 0$$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}$$

$$=8(-1)-2(-1)$$

$$=-8+2$$

$$=-6$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-\cancel{6}^{3}}{-\cancel{10}5}=\frac{3}{5}$$

$$|A_y|=\begin{vmatrix}4 & 8\\3 & -1\end{vmatrix}$$

$$=4(-1)-(3)(8)$$

$$=-4-24$$

$$=-28$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{-28}{-10}=\frac{14}{5}$$

$$S.S.=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$$

(iv) $3x-2y=-6$

$5x-2y=-10$

In matrices form

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let

$$A=\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}-6\\-10\end{bmatrix}$$

$$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$$

$=3(-2)-5(-2)$

$=-6+10$

$|A|=4\neq 0$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}-6 & -2\\-10 & -2\end{vmatrix}$$

$=-6(-2)-(-2)(-10)$

$=12-20$

$|A_x|=-8$

$$x=\frac{|A_x|}{|A|}=\frac{-\cancel{8}^2}{\cancel{4}}$$

$x=-2$

$$|A_y|=\begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$$

$=3(-10)-(5)(-6)$

$=-30+30$

$=0$

$$y=\frac{|A_y|}{|A|}=\frac{0}{4}$$

$y=0$

$S.S.=\{(-2,0)\}$

(v) $3x-2y=4$

$-6x+4y=7$

In matrices form

$$\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$

Let

$$A=\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}4\\7\end{bmatrix}$$

$$|A|=\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix}$$

$=3(4)-(-6)(-2)$

$=12-12$

$|A|=0$

As $|A|=0$, so solution is not possible

(vi) $4x+y=9$

$-3x-y=-5$

In matrices form

$$\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\-5\end{bmatrix}$$

Let

$$A=\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}9\\-5\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 1\\-3 & -1\end{vmatrix}$$

$=4(-1)-(-3)(1)$

$=-4+3$

$|A|=-1\neq 0$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}9 & 1\\-5 & -1\end{vmatrix}$$

$=9(-1)-1(-5)$

$=-4$

$$x=\frac{|A_x|}{|A|}=\frac{\not-4}{\not-1}$$

$x=4$

$$|A_y|=\begin{vmatrix}4 & 9\\ -3 & -5\end{vmatrix}$$

$=4(-5)-9(-3)$

$=-20+27$

$=7$

$$y=\frac{|A_y|}{|A|}=\frac{7}{-1}$$

$y=-7$

$S.S=\{(4,-7)\}$

(vii) $2x-2y=4$

$-5x-2y=-10$

In matrices form

$$\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -10\end{bmatrix}$$

Let

$$A=\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix};X=\begin{bmatrix}x\\ y\end{bmatrix};B=\begin{bmatrix}4\\ -10\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$

$=2(-2)-(-5)(-2)$

$=-4-10$

$|A|=-14\neq 0$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}4 & -2\\ -10 & -2\end{vmatrix}$$

$=4(-2)-(-10)(-2)$

$=-8-20$

$=-28$

$$x=\frac{|A_x|}{|A|}=\frac{\not-\not28^{\,2}}{\not-\not14}$$

$x=2$

$$|A_y|=\begin{vmatrix}2 & 4\\ -5 & -10\end{vmatrix}$$

$=2(-10)-(-5)(4)$

$=-20+20$

$=0$

$$y=\frac{|A_y|}{|A|}=\frac{0}{-14}$$

$y=0$

$S.S=\{(2,0)\}$ ans.

(viii) $3x-4y=4$

$x+2y=8$

In matrices form

$$\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 8\end{bmatrix}$$

Let

$$A=\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix};X=\begin{bmatrix}x\\ y\end{bmatrix};B=\begin{bmatrix}4\\ 8\end{bmatrix}$$

$$|A|=\begin{vmatrix}3 & -4\\ 1 & 2\end{vmatrix}$$

$=3(2)-1(-4)$

$=6+4$

$|A|=10\neq 0$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix}4 & -4\\ 8 & 2\end{vmatrix}$$

$=4(2)-8(-4)$

$=8+32$

$=40$

$$x=\frac{|A_x|}{|A|}=\frac{40\,4}{10}$$

$x=4$

$$|A_y|=\begin{vmatrix}3 & 4\\ 1 & 8\end{vmatrix}$$

$=3(8)-1(4)$

$=24-4$

$=20$

$$y=\frac{|A_y|}{|A|}=\frac{20^{2}}{10}$$

$y=2$

$S.S.=\{(4,2)\}$ ans.

Q.2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150cm. Find dimensions of the rectangle?

Let width of rectangle = x.

and length of rectangle = y

According to first condition

$y=4x$

$4x-y=0$.........(i)

According to 2nd condition

Perimeter =150cm.

$2(x+y)=150$

$x+y=\frac{150}{2}$

$x+y=75$ ....(ii)

In matrices form

$$\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}75\\ 0\end{bmatrix}$$

AX=B

$\Rightarrow$ $X=A^{-1}B$

Now

$$A=\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 1\\ 4 & -1\end{vmatrix}$$

$=1(-1)-4(1)$

$=-1-4$

$=-5\neq 0$

$$Adj\ A=\begin{bmatrix}-1 & -1\\ -4 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj}\,A$$

$$A^{-1}=-\frac{1}{5}\begin{bmatrix}-1 & -1\\ -4 & 1\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}$$

$\Rightarrow$ $X=A^{-1}B$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}\begin{bmatrix}75\\ 0\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}1(75)+1(0)\\ 4(75)+(-1)(0)\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}75\\ 300\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{75}{5}\\ \frac{300}{5}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ 60\end{bmatrix}$$

$\Rightarrow$ $x=15cm$

$\Rightarrow$ $y=60cm$

Q.3. Two sides of rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.

Let required sides of rectangle are x and y.

According to first condition

$$x - y = 3.5 \longrightarrow (i)$$

According to 2[nd] condition

Perimeter =67

2(x+y) =67

$$\Rightarrow \quad x + y = 33.5 \longrightarrow (ii)$$

In matrices form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

We have

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}; A_x = \begin{bmatrix} 3.5 & -1 \\ 33.5 & 1 \end{bmatrix},$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 1 & 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(1) - 1(-1)$$

$$= 1 + 1 = 2 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{\begin{vmatrix} 3.5 & -1 \\ 33.5 & 1 \end{vmatrix}}{2}$$

$$= \frac{3.5(1) - 33.5(-1)}{2}$$

$$= \frac{3.5 + 33.5}{2}$$

$$= \frac{37}{2} = 18.5$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{\begin{vmatrix} 1 & 3.5 \\ 1 & 33.5 \end{vmatrix}}{2}$$

$$= \frac{1(33.5) - 1(3.5)}{2}$$

$$= \frac{33.5 - 3.5}{2}$$

$$= \frac{30}{2} = 15$$

$$\Rightarrow \quad x = 18.5, \qquad y = 15$$

Q.4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Let third angle of triangle = y

and two equal angle of triangle = x

we know that

$$x + x + y = 180^o$$

$$2x + y = 180^o \ldots\ldots\ldots(i)$$

According to given condition.

$$y = 2x - 16$$

$$2x - y = 16$$

In matrices form

$$\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 16 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow \quad X = A^{-1}B$$

Now

$$A = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}$$

$$|A| = 2(-1) - 2(1)$$

$$= -2 - 2$$
$$= -4 \neq 0$$
$$\text{Adj} A = \begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$
$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$
$$= \frac{1}{-4}\begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$
$$\Rightarrow \quad X = A^{-1}B$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 180 \\ 16 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} 1(180)+1(16) \\ 2(180)+(-2)(16) \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} 180+16 \\ 360-32 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} 196 \\ 328 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

Hence: $x = 49^\circ$, $y = 82^\circ$

Required angles are 49°, 49°, 82°.

Q.5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle?

Let acute angles of right angled triangle are x and y

We know that

$$x + y = 90^o \ (i)$$

According to given condition

$$x = 2y + 12^o$$
$$x - 2y = 12^o \longrightarrow (ii)$$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 12 \end{bmatrix}$$

We have

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}, \ A_x = \begin{bmatrix} 90 & 1 \\ 12 & -2 \end{bmatrix}$$
$$A_y = \begin{bmatrix} 1 & 90 \\ 1 & 12 \end{bmatrix}$$

Now $$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$
$$|A| = 1(-2) - 1(1)$$
$$= -2 - 1$$
$$= -3 \neq 0$$
$$x = \frac{|A_x|}{|A|}$$
$$= \frac{\begin{vmatrix} 90 & 1 \\ 12 & -2 \end{vmatrix}}{-3}$$
$$= \frac{90(-2) - 1(12)}{-3}$$
$$x = \frac{-180 - 12}{-3}$$
$$= \frac{-192}{-3} = 64^o$$
$$y = \frac{|A_y|}{|A|}$$

$$y=\frac{\begin{vmatrix}1 & 90\\ 1 & 12\end{vmatrix}}{-3}$$

$$=\frac{1(12)-1(90)}{-3}$$

$$=\frac{12-90}{-3}$$

$$=\frac{-78}{-3}$$

$$=26^o$$

$\therefore$ Required angles are 26° and 64°

$\Rightarrow$ $x=64^o$

$\Rightarrow$ $y=26^o$

Q6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Let required speed of two cars are x and y

According to given condition

$$x-y=6$$

$$\frac{9}{2}x-\frac{9}{2}y=600-123=477$$

$$x-y=6$$

$$9x+9y=477\times 2=954$$

$\Rightarrow$
$$x-y=6$$
$$9x+9y=954$$

In matrix form

$$\begin{bmatrix}1 & -1\\ 9 & 9\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}6\\ 954\end{bmatrix}$$

$$A=\begin{bmatrix}1 & -1\\ 9 & 9\end{bmatrix}, A_x=\begin{bmatrix}6 & -1\\ 954 & 9\end{bmatrix}$$

$$A_y=\begin{bmatrix}1 & 6\\ 9 & 954\end{bmatrix}$$

Now

$$A=\begin{bmatrix}1 & -1\\ 9 & 9\end{bmatrix}$$

$$|A|=1(9)-(-1)(9)$$
$$=9+9=0$$
$$=18\neq 0$$

$$x=\frac{|A_x|}{|A|}=\frac{\begin{vmatrix}6 & -1\\ 954 & 9\end{vmatrix}}{18}$$

$$=\frac{6(9)-(-1)(954)}{18}=\frac{54+954}{18}=\frac{1008}{18}=56km/h$$

$$y=\frac{|A_y|}{|A|}=\frac{\begin{vmatrix}1 & 6\\ 9 & 954\end{vmatrix}}{18}$$

$$=\frac{1(954)-6(9)}{18}$$

$$=\frac{954-54}{18}$$

$$=\frac{900}{18}=50km/h$$

OBJECTIVE

1 .The order of matrix [2 1] is ....

(a) 2-by-1 (b) 1-by-2

(c) 1-by-1 (d) 2-by-2

2. $\begin{bmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{bmatrix}$ is called .... Matrix.

(a) zero (b) unit

(c) scalar (d) singular

3. Which is order of a square matrix ?

(a) 2-by-2 (b) 1-by-2

(c) 2-by-1 (d) 3-by-2

4. Which is order of a rectangular matrix?

(a) 2-by-2 (b) 4-by-4

(c) 2-by-1 (d) 3-by-3

5. Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is ...:

(a) 3-by-2 (b) 2-by-3

(c) 1-by-3 (d) 3-by-1

6. Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is ....

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

7. If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix} = 0$, then x is equal to:

(a) 9 (b) –6

(c) 6 (d) –9

8. Product of $[x \quad y]\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is ....

(a) $[2x+y]$ (b) $[x-2y]$

(c) $[2x-y]$ (d) $[x+2y]$

9. If $x + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then x is equal to.........

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

10. The idea of a matrices was given by:__

(a) Arthur Cayley (b) Dr. Aslam

(c) Dr. Ali (d) Dr. Khalid

11. The matrix $M = [2 \ -1 \ 7]$ is a---- matrix.

(a) Row (b) Column

(c) Square (d) Null

12. The matrix $N = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ is a ____ matrix.

(a) Row (b) Column

(c) Square (d) Null

13. The matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$ is a __ matrix.

(a) Rectangular (b) Square

(c) Row (d) Column

14. The matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$ is a __ matrix.

(a) Rectangular (b) Square

(c) Row (d) Column

15. If A is a matrix then its transpose is denoted by:

(a) A^c (b) A^t

(c) A (d) $(A^t)^t$

16. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ then $-A =$ ______

(a) $\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$

17. A square matrix is symmetric if ____

(a) $A^t = A$ (b) $A^c = A$

(c) $(A^t)^t = -A^t$ (d) None

18. A square matrix is skew-symmetric if:

(a) $A^t = -A$ (b) $A^c = -A$

(c) $(A^t)^t = -A^t$ (d) None

19. The matrix $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is a __ matrix.

(a) Diagonal (b) Scalar

(c) Identity (d) Zero

20. The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is a__matrix.

(a) Diagonal (b) Scalar
(c) Identity (d) Zero

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a ____ matrix.

(a) Diagonal (b) Identity
(c) Zero (d) None

22. The scalar matrix and identity matrix are ____ matrices.

(a) Diagonal (b) Rectangular
(c) Zero (d) None

23. Every diagonal matrix is not a ____ matrix.

(a) Scalar (b) Identity
(c) Scalar or identity (d) None

24. If A, B are two matrices and A^t, B^t are their respective transpose, then:

(a) $(AB)^t = B^t A^t$ (b) $(AB)^t = A^t B^t$
(c) $A^t B^t = AB$ (d) None

25. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A is:

(a) ad – bc (b) bc – ad
(c) ad + bc (d) bc + ad

26. A square matrix A is called singular if

(a) $|A| \neq 0$ (b) $|A| = 0$
(c) $A = 0$ (d) $A^t = 0$

27. A square matrix A is called non-singular if:

(a) $|A| = 0$ (b) $A = 0$
(c) $|A| \neq 0$ (d) $A^t = 0$

28. Inverse of identity matrix is ____ matrix.

(a) Identity (b) Zero
(c) Rectangular (d) None

29. $AA^{-1} = A^{-1}A =$ ______

(a) Identity matrix
(b) Rectangular matrix
(c) Zero matrix (d) none

30. $(AB)^{-1} =$ ____

(a) $A^{-1} B^{-1}$ (b) $B^{-1} A^{-1}$
(c) BA (d) AB

31. Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is ____

(a) $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

Answer Key

1	b	**2**	c	**3**	a	**4**	c	**5**	d
6	a	**7**	a	**8**	c	**9**	d	**10**	a
11	a	**12**	b	**13**	a	**14**	b	**15**	b
16	a	**17**	a	**18**	a	**19**	a	**20**	b
21	b	**22**	a	**23**	c	**24**	a	**25**	a
26	b	**27**	c	**28**	a	**29**	a	**30**	b
31	a								

2. Complete the following:

i. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called …… matrix.

Null / Zero matrix

ii. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ……. Matrix.

Identity /Unit matrix

iii. Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is …

$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

iv. In matrix multiplication, in general, AB …… BA.

$\neq$

v. Matrix A + B may be found if order of A and B is ……

Same

vi. A matrix is called …. matrix if number of rows and columns are equal.

Square

3. If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, **then find a and b.**

Ans. $\Rightarrow$ $a + 3 = -3$ ……(I)

$b - 1 = 2$ ……..(II)

From (I) $a = -3 - 3$

$a = -6$

From (II) $b = 2 + 1$

$b = 3$

4. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, **then find the following.**

Ans.

(i) 2A + 3B

$$2A + 3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$$

(ii) $-3A + 2B = -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix}$$

(iii) $-3\,(A+2B)$

$$A + 2B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix} = \begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$-3(A+2B) = -3\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -36 & 15 \\ -9 & 6 \end{bmatrix}$$

(iv) $\frac{2}{3}(2A - 3B)$

$$2A - 3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-15 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$\frac{2}{3}(2A-3B) = \frac{2}{3}\begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-22}{3} & \frac{36}{3} \\ \frac{16}{3} & \frac{6}{3} \end{bmatrix} = \begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix}$$

5. Find the value of x, if

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + x = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}.$$

Ans. $\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + x = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$

$$x = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2+3 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$,

then prove that

i) $AB \neq BA$

Ans. $AB \neq BA$

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(-3)+1(5) & 0(4)+1(-2) \\ 2(-3)+-3(5) & 2(4)+-3(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -3(0)+4(2) & -3(1)+4(-3) \\ 5(0)+-2(2) & 5(1)+-2(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix}$$

$$AB \neq BA$$

7. If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ **and**

$B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$, **then verify that**

(i) $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1} A^{-1}$

Ans. (i) $(AB)^t = B^t A^t$

L.H.S = $(AB)^t$

$$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3(2)+2(-3) & 3(4)+2(-5) \\ 1(2)+-1(-3) & 1(4)+-1(-5) \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix}$$

R.H.S = $B^t A^t$

$$A^t = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3)+-3(2) & 2(1)+-3(-1) \\ 4(3)+-5(2) & 4(1)+-5(-1) \end{bmatrix}$$

$$=\begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix}$$

L.H.S = R.H.S

Hence: $(AB)^t = B^t A^t$

(ii) $\mathbf{(AB)^{-1} = B^{-1} A^{-1}}$

$$A=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}, B=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

L.H.S = $(AB)^{-1}$

$$AB=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$=\begin{bmatrix} 3(2)+2(-3) & 3(4)+2(-5) \\ 1(2)+-1(-3) & 1(4)-1(-5) \end{bmatrix}$$

$$=\begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\operatorname{Adj} AB$$

$$|AB|=\begin{vmatrix} 0 & 2 \\ 5 & 9 \end{vmatrix}=0(9)-5(2)=-10\neq 0$$

$$(AB)^{-1}=\frac{1}{-10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}=\frac{1}{10}\begin{bmatrix} -9 & 2 \\ 5 & 0 \end{bmatrix}=\begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix}$$

R.H.S = $B^{-1} A^{-1}$

$$A=\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A|=3(-1)-1(2)=-3-2=-5\neq 0$$

$$\operatorname{Adj} A=\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\operatorname{Adj} A=\frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$B=\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$|B|=2(-5)-(-3)(4)$$

$$=-10+12=2\neq 0$$

$$B^{-1}=\frac{1}{|B|}\operatorname{Adj} B$$

$$=\frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1}A^{-1}=\left(-\frac{1}{5}\right)\left(\frac{1}{2}\right)\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$=\frac{1}{-10}\begin{bmatrix} -5(-1)+-4(-1) & -5(-2)+-4(3) \\ 3(-1)+2(-1) & 3(-2)+2(3) \end{bmatrix}$$

$$=\frac{-1}{10}\begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}=\frac{1}{-10}\begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-9}{10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix}$$

L.H.S = R.H.S.

Hence: $(AB)^{-1}=B^{-1}A^{-1}$